

Structural Reliability Analysis Based on Augmented Multiplier Method and Bat Algorithm

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Abstract

This paper puts forward a kind of reliability analysis method based on augmented multiplier method and the bat algorithm. The minimum reliability index as the objective function, the method to affect the structural reliability index of random variables constitute the constraint condition of limit state equation of structural reliability optimization mathematical model. The present method uses the augmented multiplier method to convert a constrained optimization problem to unconstrained optimization problem, and at last use the bat algorithm (BA) optimization for the calculation of reliability index. The stability and effectiveness of the proposed algorithm are verified by numerical examples and engineering examples, and the influence of parameter uncertainty on the reliability index is discussed.

Keywords

The augmented multiplier method; Bat algorithm; Structural reliability optimization design; Uncertainty.

1. Introduction

In engineering practice, due to the influence of various uncertain factors such as product material properties, machining accuracy and assembly error, the actual parameters and design parameters of the structure will produce errors. In order to ensure the safety and effectiveness of the structure, it is necessary to conduct reliability analysis on the structure to pre-evaluate the failure risk of the structure [1]. Ashok et al. [2] proposed binary differential evolution algorithm (BDE) for reliability index analysis of composite power system, and applied this search method to RBTS and IEEE-RTS test system, which can analyze the reliability of a small number of system states. Jiang et al. [3] proposed a time-varying analysis method for reliability calculation problems with different distribution types and interval parameter uncertainties, which effectively solved similar problems. Zadehet al. [4] proposed a metamodel-based optimization structure design method to solve multi-disciplinary and multi-objective optimization design problems. By introducing SQP method and metamodel, fuzzy logic decisions located at the Pareto solution can be made. Wei et al. [5] studied the interval motion accuracy reliability of the manipulator based on Chebyshev inclusion polynomials, and calculated and evaluated the motion accuracy reliability of the space manipulator based on the mechanism reliability analysis method and interval uncertainty model. Tang [6] solved the problem of interval uncertainty in the reliability analysis of machine parts by using a reliability analysis method of global sensitivity based on partial elasticity theory. Liu [7] proposed a new method of uncertain reliability analysis with probability and interval mixing, which described the uncertain variables with sufficient sample information as random variables and expressed

the uncertain variables with interval in the lack of sample information in the structural system. However, due to the increasingly complex structural functional functions, traditional reliability analysis methods are difficult to meet the solving needs, and system reliability problems usually have a large number of uncertain parameters. If the parameter uncertainty is ignored, it will bring errors to the calculation of reliability indicators.

Based on this, a structural reliability analysis method based on augmented multiplier method and bat algorithm is proposed. The method takes the minimum reliability index as the objective function, and the limit state equation composed of random variables that affect the reliability index as the constraint condition, establishes a mathematical model, conducts the structural reliability analysis, and discusses the influence of parameter uncertainty on the reliability index.

2. The Establishment of Reliability Index

Suppose $X_1, X_2, \dots, X_n$ is the basic random variable affecting the structural function, then the functional function of the structure can be written

$$Z = g(X_1, X_2, \dots, X_n) \quad (1)$$

When $Z > 0$, indicates that the structure is in a reliable state; $Z < 0$, indicates that the structure is in the failure state; $Z = 0$, Indicates that the structure is in the limit state plane. Accordingly, the limit state equation of the structure can be constructed as follows

$$Z = g(X_1, X_2, \dots, X_n) = 0 \quad (2)$$

The non-normal variables are normalized by Rackwitz-Fiessler method, and the mean μ'_{xi} , standard deviation σ'_{xi} and reliable index of β the normal distribution are obtained

$$\mu'_{xi} = X_i^* - \Phi^{-1}[F_{xi}(x_i^*)] \cdot \sigma'_{xi} \quad (3)$$

$$\sigma'_{xi} = \phi\{\Phi^{-1}[F_{xi}(x_i^*)]\} / f_{xi}(X_i^*) \quad (4)$$

$$\beta = \left\{ \sum_{i=1}^n [(X_i^* - \mu'_{xi}) / \sigma'_{xi}]^2 \right\}^{1/2} \quad (5)$$

Since the checking point is unknown, β can be regarded as the function of the limit state surface point $p(X_1, X_2, \dots, X_n)$, and the minimum value of β can be found by solving, namely, the reliable index β and the checking point $p^*(X_1^*, X_2^*, \dots, X_n^*)$ can be obtained. From the above, the following optimization mathematical model can be established

$$\min \quad \beta^2 = \sum_{i=1}^n [(X_i^* - \mu'_{xi}) / \sigma'_{xi}]^2 \quad (6)$$

$$s.t \quad Z = g(X_1^*, X_2^*, \dots, X_n^*) = 0 \quad (7)$$

The equation constrained augmented multiplier method is used to transform the above constrained mathematical model into an unconstrained mathematical model, and the formula is as follows

$$\min M_{\lambda}(X^*, r, \lambda) = M(X^*) + \frac{r}{2}[g(X^*)]^2 + \lambda g(X^*) \quad (8)$$

In the formula, $M(X^*)$ is β^2 . The second item at the right end is the penalty item, where β^2 is the penalty factor. The third term on the right is the multiplier, and λ is the Lagrange multiplier. When the augmented multiplier method is used, the penalty factor only needs to take a larger value, and it is not required that the penalty factor tends to infinity. In this way, the problem of selecting the penalty factor using the penalty function method alone is avoided.

3. The Bat Algorithm

BAT Algorithm (BA) is a heuristic search Algorithm proposed by Yang based on swarm intelligence in 2010. It is an effective method to search for the global optimal solution [8]. Many studies have shown that in the process of searching for prey, bats search for prey in a wide range by adjusting the intensity of ultrasonic pulse. When approaching prey, bats will gradually increase the frequency of ultrasonic pulse and decrease the intensity, so as to accurately grasp the spatial position of prey. On this basis, the bat algorithm is evolved, which is an optimization particle swarm optimization algorithm based on iteration. Firstly, a set of random solutions are initialized arbitrarily in the feasible space, and then, an iterative algorithm is used to search for the optimal solution, and random new solutions are generated around the optimal solution to enhance local search, so as to accurately grasp the spatial position of the target.

3.1. Location Update

According to the process of the bat foraging, the bat's speed, frequency and coordinate position can be expressed by the following formula

$$v_i^{t+1} = v_i^t + (x_{best} - x_i^t) f_i \quad (9)$$

$$f_i = f_{min} + (f_{max} - f_{min}) \beta \quad (10)$$

$$x_i^{t+1} = x_i^t + v_i^{t+1} \quad (11)$$

In the formula, $\beta \in [0,1]$ is a random number, f_{min} and f_{max} are the range of frequency f , and x_{best} is the global optimal solution at the moment t .

A bat is in a viable space. Once the current optimal solution is determined, each bat performs a random motion, resulting in a new set of solutions, with the new positions represented by the following equation

$$x_{new} = x_{old} + \varepsilon A^t \quad (12)$$

Where, $\varepsilon \in [0,1]$ is the random number and A^t is the mean value of the pulse intensity of the whole bat population at time t .

3.2. Pulse Intensity and Frequency

Many studies showed that the bats initially sent out strong ultrasonic pulses to achieve a wide range of search, and as they approached the prey, they reduced the intensity of the ultrasonic pulses and increased the frequency of the transmissions to achieve precise positioning. The intensity and frequency of the ultrasonic pulse emitted by each bat are constantly updated with the iteration of the algorithm. The following equation is used to simulate this process.

$$A_i^{t+1} = \alpha A_i^t \quad (13)$$

$$r_i^{t+1} = r_i^0 (1 - e^{-\gamma t}) \quad (14)$$

Where, $0 < \alpha < 1, \gamma > 0$; when, $t \rightarrow \infty, A_i^t \rightarrow 0, r_i^t \rightarrow r_i^0$.

4. Example

4.1. Numerical Example

Suppose that variables x_1 and x_2 are normally distributed, with mean values of 10 and 2.5 and standard deviations of 2 and 0.375, respectively, and the limit state equation is given as follows

$$g = 18.46 - 7.48x_1x_2^{-3} = 0 \quad (15)$$

The BA method, the first order second moment method (FOSM) and the Monte Carlo method (MCS) are respectively used to solve the reliability index, and the calculated results of the reliability index obtained are shown in Table 1. Iterative comparison of reliability indexes between BA method and FOSM method is shown in Figure 1.

Table 1. Reliability index calculation results of the three methods.

Methods	Reliability index	Failure probability	Iterative times
MCS	2.344	0.0095	107
FOSM	2.330	0.0099	12
BA	2.335	0.0097	3

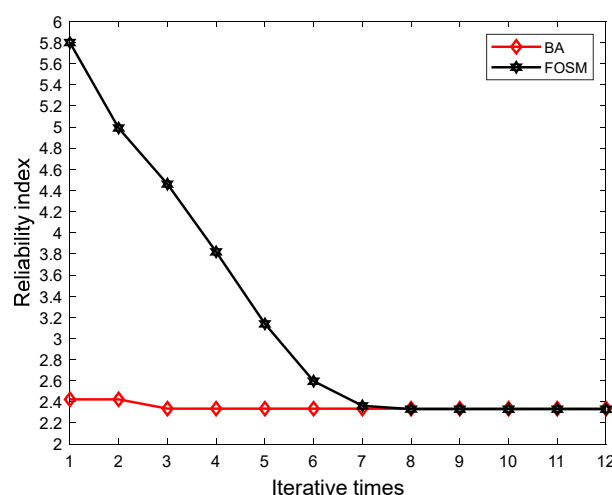


Figure 1. Shows the iterative comparison of reliability index between BA method and FOSM method

As can be seen from Table 1, the reliability index calculated by the present algorithm in this paper is 2.335. The reliable index calculated by FOSM method is 2.330. The reliability index calculated by MCS method is 2.344, and the reliability index calculated by the three algorithms is roughly the same, which verifies the feasibility of the present algorithm in this paper. Meanwhile, the proposed algorithm in this paper only needs to iterate 3 times to obtain the reliability index, while the FOSM method needs to iterate 12 times. As can be seen from Figure 1, the convergence speed of the present algorithm in this paper is faster and more stable, so the present algorithm in this paper is better.

4.2. Engineering Example

In recent years, people's design requirements for body crashworthiness have been continuously improved. Under the condition of side impact, the maximum acceleration of B-pillar is an important index to measure body crashworthiness [9]. Consider the car collision problem shown in Figure 2, where the movable barrier hits the car from the side at a speed of 50km/h.

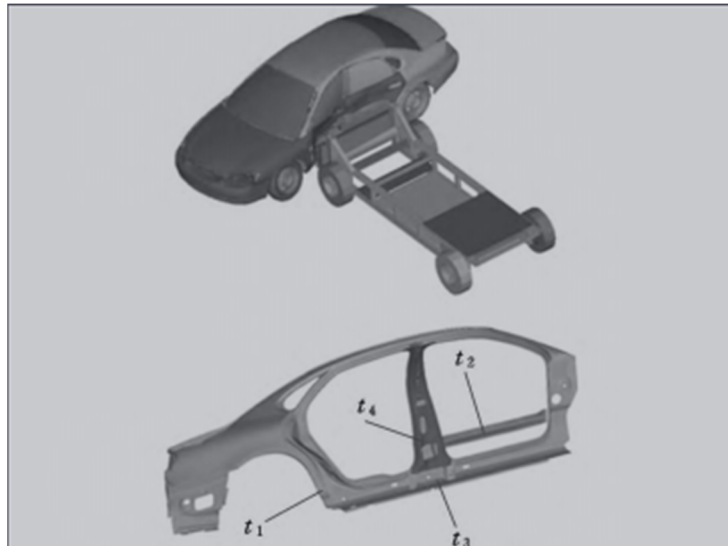


Figure 2. Finite element model of vehicle side impact

The four body plate thicknesses shown in Figure 2 are selected as the key design variable $t=(t_1, t_2, t_3, t_4)$ for analysis. In order to meet the requirements of side impact crashworthiness, the maximum acceleration of B-pillar a_0 shall not exceed the allowable value. The four variables are all random variables, which are normally distributed, and their mean values are $\mu t_1=0.9mm$, $\mu t_2=2.1mm$, $\mu t_3=1.0mm$ and $\mu t_4=1.4mm$, and standard deviations are $\sigma t_1=0.09mm$, $\sigma t_2=0.42mm$, $\sigma t_3=0.14mm$ and $\sigma t_4=0.2mm$. The optimal Latin hypercube design method was used to select 41 sample points in the design space to construct the response surface of functional functions, whose functional functions can be expressed as.

$$g(t)=a_0 - a(t_1, t_2, t_3, t_4) \quad (16)$$

Using the proposed algorithm, the reliability index $\beta=9.3581$ and failure probability $P_f=4.05^{-21}$ are obtained, and it can be seen that the failure probability is close to 0, which indicates that the calculation example has high reliability under the conditions presented in this paper.

In structural reliability analysis and design, due to the actual situation and cost factors and other limitations, there is often a lack of accurate sample information to describe the system, resulting in a variety of uncertain factors [10]. The influence of the uncertainty of mean and standard deviation on the calculation results of reliable indicators is discussed in the following engineering examples.

(1) If the uncertainty of the mean value of the variable is considered and the standard deviation is guaranteed to remain unchanged, the variation range and uncertainty of the reliable index calculated by using the method in this paper are shown in Table 2.

Table 2. Reliable indicators considering mean uncertainty

Mean uncertainty	Reliable index range	Uncertainty of reliable indicators
2%	[9.736,10.034]	3.39%
4%	[9.496,10.742]	6.16%
6%	[9.264,11.532]	10.91%
8%	[8.968,12.508]	16.49%

It can be seen from Table 2 that the corresponding relationship between mean uncertainty and reliability index uncertainty is shown in Figure 3.

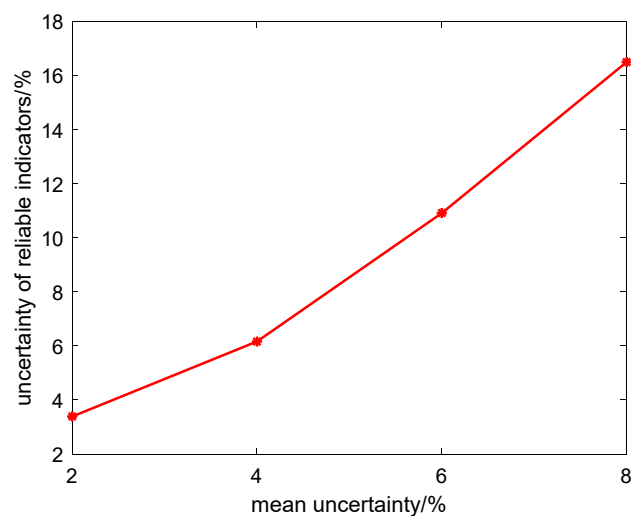


Figure 3. The influence of mean uncertainty on reliability index

As can be seen from Table 2 and Figure 3, with the constant change of mean uncertainty, the uncertainty of its reliability index also changes, and there is a positive correlation.

(2) If the uncertainty of standard deviation is taken into account and the mean value remains unchanged, the variation range and uncertainty of reliable indicators calculated by using the present method in this paper are shown in Table 3.

Table 3. Reliable indicators considering the uncertainty of standard deviation

Standard deviation uncertainty	Reliable index range	Uncertainty of reliable indicators
2%	[9.808,10.537]	3.58%
4%	[9.606,10.747]	5.61%
6%	[9.358,11.043]	8.26%
8%	[8.952,11.224]	11.26%

It can be seen from Table 3 that the corresponding relationship between standard deviation uncertainty and reliability index uncertainty is shown in Figure 4.

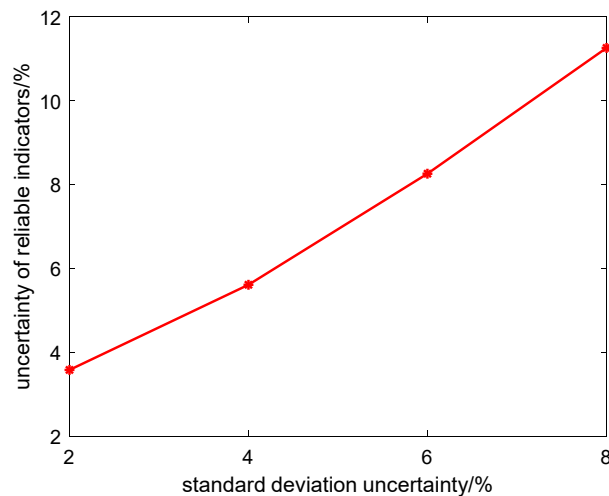


Figure 4. Influence of standard deviation uncertainty on reliability index

As can be seen from Table 3 and Figure 4, with the constant change of mean uncertainty, the uncertainty of its reliability index also changes, and there is a positive correlation.

5. Conclusion

In this paper, a structural reliability analysis method based on augmented multiplier method and bat algorithm is proposed, which provides an effective tool for solving the structural reliability optimization design problems with considering uncertainties. The effectiveness of the proposed method is verified by two numerical examples. The results show that the proposed method can deal with structural functional functions with a certain degree of nonlinearity, and the number of iterations is less than that of the traditional algorithm while ensuring the accuracy. Mean uncertainty and standard deviation uncertainty are linearly correlated with the reliability index uncertainty, and mean uncertainty has more influence on the calculation of reliability index than standard deviation uncertainty.

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References

- [1] N.M.Okasha, An improved weighted average simulation approach for solving reliability-based analysis and design optimization problems, *Structural Safety* 60 (2016) 47-55.
- [2] B.Ashok, Differential evaluation based intelligent system state search method for composite power system reliability evaluation, *Journal of The Institution of Engineers (India): Series B* 96 (3) (2015) 237-244.
- [3] C. Jiang, X.P. Huang, X. Han, et al, Structural Time-varying Reliability Analysis Method with Interval Uncertainty, *Chinese Journal of Mechanical Engineering* 49 (10) (2013) 186-193. (In Chinese)
- [4] P.M. Zadeh, M. Sayadi, A. Kosari, An efficient metamodel-based multi-objective multidisciplinary design optimization framework, *Applied Soft Computing Journal* 74 (2019) 760-782.

- [5] J. Wei, Interval Motion Accuracy Reliability Analysis of Manipulators Based on Chebyshev Inclusion Polynomial, Journal of Harbin Institute of Technology (2018) 1005-9113. (In Chinese)
- [6] Z.Tang, A Sensitivity and Reliability Analysis Method for Interval Uncertain Parameters, Computer Integrated Manufacturing Systems 23 (12) (2017) 2593-2603. (In Chinese)
- [7] H.B. Liu, Reliability analysis method of system with probabilistic and interval mixed uncertainty, Chinese Journal of Mechanics 49 (2) (2017) 456-466. (In Chinese)
- [8] X.S. He, W.J. Ding, X.S. Yang, Bat Optimization Algorithm Based on Simulated Annealing Gaussian Perturbation, Computer application research 31 (02) (2014) 392-397. (In Chinese)
- [9] C.Jiang, Q.Q. Deng, W. Zhang, Structural Second-order Reliability Analysis Method Considering Parameter Correlation, China Mechanical Engineering 27 (22) (2016) 3068-3074+3081. (In Chinese)
- [10] N.C. Xiao, C.N. Zhou, C.L. Zhang, Z.L. Liu, Structural Reliability Analysis Method with Mixed Uncertainties, Journal of University of Electronic Science and Technology of China 47(05) (2018) 788-792. (In Chinese)